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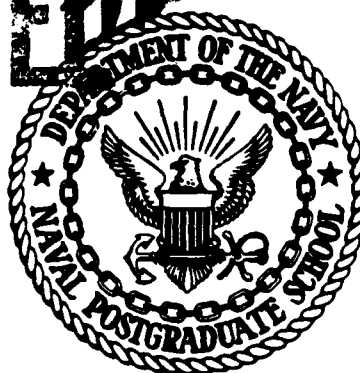
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Seasonal Production and Sales Planning  
with Limited Shared Tooling  
at the Key Operation

by

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A. M. Geoffrion  
G. H. Bradley

December 1979

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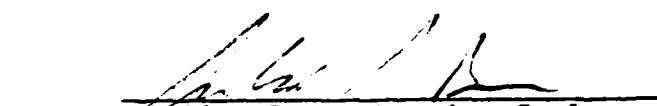
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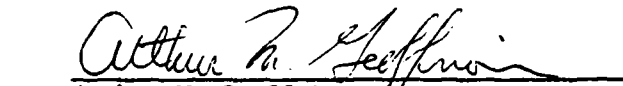
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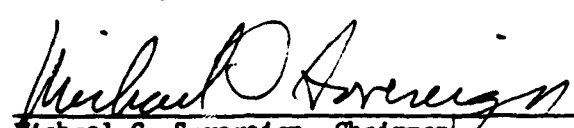
  
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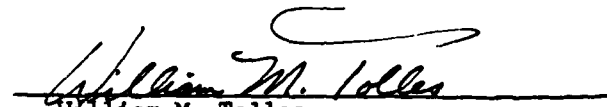
  
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SEASONAL PRODUCTION AND SALES PLANNING  
WITH LIMITED SHARED TOOLING AT THE KEY OPERATION

by

Gerald G. Brown  
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Gordon H. Bradley

ABSTRACT

Lagrangean relaxation is applied to a class of very large mixed integer linear programming problems representing seasonal production and sales planning in a situation where limited tooling is available at the key production operation. A successful application to the injection molding industry is described.

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This paper addresses production and sales planning in a seasonal industry with a single dominant production operation for which tooling can be shared among parts and is limited in availability.

The specific context of our experience is the production of injection molded plastic pipes and fittings destined for the building and chemical industries. The dominant production operation is injection molding and the tooling consists of mold bases used to adapt the injection molding machines to the molds proper. Mold bases typically require 4-6 calendar months to obtain at a cost which can approach the cost of the molding machine itself so their availability is limited and good utilization is important.

Dominant production operations with limited and possibly shared tooling arise in many other contexts. Likely candidates include production facilities based on casting, molding, stamping, extrusion, or pressing of finished or nearly finished products. Dies and molds and associated adaptive tooling are usually expensive and often designed for use with more than one end product. Machine tooling with elaborate jigs and fixtures constitutes another large area of potential application.

An informal statement of the problem treated is as follows. A facility produces many different parts (products), each by a single operation calling for a specific type of tool and any one of a number of machines compatible with the tool. Machines are aggregated into machine groups and tools into tool types. Production and sales are to be planned for each part over a multiperiod horizon (typically monthly for a full year):

*Determine*

- . how much of each part to produce in each time period
- . how much of each part to sell in each time period
- . how much of each part to carry forward as inventory from each time period into the next

- . a tool/machine assignment schedule specifying, for each time period, the number of days of production of each tool type in conjunction with each compatible machine group

*so as to satisfy all necessary constraints*

- . limited availability of tools in each time period
- . limited availability of machines in each time period
- . tool/machine compatibility restrictions
- . for each part in each time period, sales cannot exceed forecast demand

*and so as to satisfy desired managerial policy constraints*

- . for each part in each time period, sales must exceed a certain fraction of demand stipulated by management
- . for each part, the ending inventory at the conclusion of the planning horizon must take on a stipulated value
- . no planned backlogging of unfilled demand

*in such a manner as to maximize total profits over all parts*

*for the duration of the planning horizon, calculated according to*

- . incremental net profit contribution per unit produced
- . less variable operating costs associated with production (by tool type and machine group)
- . less fixed costs associated with production (by part, for each period with positive production)
- . less inventory holding costs.

The problem as stated has elements in common with many familiar dynamic planning and resource allocation problems. It is more detailed than most seasonal planning problems in that discrete fixed costs are included and no aggregation is necessary over parts, yet it stops short of encompassing detailed scheduling because other aggregations are employed (tools  $\rightarrow$  tool types, machines  $\rightarrow$  machine groups, time  $\rightarrow$  time periods). Related production planning and scheduling problems in the molding industry can be found in [3] [6] [7].

A proper mathematical formulation as a mixed integer linear program is given in Sec. 1. The next section presents a solution approach based on a

particularly attractive Lagrangean relaxation and sketches our full scale computational implementation. Sec. 3 describes computational experience with the injection molding application mentioned earlier. For this application, solutions well within 2% of optimum are routinely produced in about 3 minutes of IBM 370/168 time for mixed integer linear programs on the order of 12,000 binary variables, 40,000 continuous variables, and 26,000 constraints.



## 1. THE MODEL

This section formally defines and discusses the model as a mixed integer linear programming problem.

The formulation makes essential use of the concept of a standard day, which is a part-specific measure of quantity. It is, for a given part, the quantity that would be produced in one calendar day if a tool of the required type were operating normally on any compatible machine.

### Notation

#### Indices

|        |   |
|--------|---|
| $i$    | indexes parts   |
| $j$    | indexes tool types  |
| $k$    | indexes machine groups  |
| $t$    | indexes time periods, $t = 1, \dots, T$                       |
| $I(j)$ | index set of the parts requiring tool type $j$                |
| $K(j)$ | index set of the machine groups compatible with tool type $j$ |

#### Given Data

|               |  |
|---------------|--|
| $a_{jt}$      | days of availability of type $j$ tools during period $t$   |
| $b_{kt}$      | days of availability of machine group $k$ during period $t$  |
| $c_{jkt}$     | variable daily operating cost during period $t$ of tool type $j$ on machine group $k$ , for compatible combinations of $j$ and $k$ |
| $d_{it}$      | demand forecast for part $i$ in period $t$ , in standard days  |
| $f_{it}$      | fixed cost associated with the production of part $i$ in period $t$  |
| $h_{it}$      | holding cost for one standard day of part $i$ held for the duration of period $t$  |
| $I_{i0}$      | initial inventory in period 1 of part $i$ , in standard days (must be $\geq 0$ )   |
| $I_{iT}$      | ending inventory desired for part $i$ , in standard days, at the conclusion of the last period (must be $\geq 0$ )                 |
| $m_{it}$      | maximum possible production of part $i$ in period $t$ , in standard days   |
| $p_i$         | profit contribution associated with one standard day's worth of part $i$ , exclusive of the other costs included in the model      |
| $\alpha_{it}$ | minimum fraction of $d_{it}$ which must be satisfied as a matter of marketing policy   |

### Decision Variables

- $I_{it}$  planned inventory of part  $i$  at the conclusion of period  $t$ , in standard days ( $1 \leq t < T$ )  
 $S_{it}$  planned sales of part  $i$  in period  $t$ , in standard days  
 $W_{jkt}$  planned production days for tool type  $j$  on machine group  $k$  during period  $t$ , for compatible combinations of  $j$  and  $k$   
 $X_{it}$  planned production of part  $i$  during period  $t$ , in standard days  
 $Y_{it}$  a binary variable indicating whether or not part  $i$  is produced during period  $t$

### Mixed Integer Linear Program

$$\begin{aligned}
 \text{MAXIMIZE} \quad & \sum_i p_i \sum_t X_{it} - \sum_j \sum_k \sum_t c_{jkt} W_{jkt} \\
 \text{I, S, W, X, Y} \quad & - \sum_i \sum_t h_{it} (I_{i,t-1} + I_{i,t})/2 - \sum_i \sum_t f_{it} Y_{it}
 \end{aligned}
 \tag{1}$$

subject to

- $$\begin{aligned}
 (2) \quad & \sum_{k \in K(j)} W_{jkt} \leq a_{jt} && , \text{ all } jt \\
 (3) \quad & \sum_{k \in K(j)} W_{jkt} = \sum_{i \in I(j)} X_{it} && , \text{ all } jt \\
 (4) \quad & \sum_j W_{jkt} \leq b_{kt} && , \text{ all } kt \\
 (5) \quad & I_{it} = I_{i,t-1} + X_{it} - S_{it} && , \text{ all } it \\
 (6) \quad & \alpha_{it} d_{it} \leq S_{it} \leq d_{it} && , \text{ all } it \\
 (7) \quad & 0 \leq X_{it} \leq m_{it} Y_{it} && , \text{ all } it \\
 (8) \quad & W_{jkt} \geq 0 && , \text{ all } jkt \\
 (9) \quad & I_{it} \geq 0 && , \text{ all } it \ (1 \leq t < T) \\
 (10) \quad & Y_{it} = 0 \text{ or } 1 && , \text{ all } it
 \end{aligned}$$

It is understood that any summations or constraint enumerations involving  $j$  and  $k$  together will run only over compatible combinations of  $j$  and  $k$ .

The objective function (1) is essentially the profit over the duration of the planning horizon. It is the profit contribution associated with production over the planning horizon, less: machine operating costs, inventory holding costs (applied to a simple 2-point estimate of the average inventory level of each part in each period), and fixed costs.

Constraints (2) and (4) respectively enforce availability limitations on tools (by type) and machines (by group). Constraint (3), a work balance on tools, relates the X's to the W's. Constraint (5) defines ending inventories in the standard way. Constraint (6) requires the planned sales to be between forecast demand and some specified fraction thereof. Constraint (7) keeps production within possible limits and also forces  $Y_{it}$  to be 1 when  $X_{it}$  is positive. Constraint (9) specifies that there be no planned backlogging. Constraints (8) and (10) require no comment.

#### Further Discussion

Some additional comments are appropriate.

1. There can be more than one tool (resp. machine) available of a given type (resp. group). Such census information, along with downtime estimates, determines the  $a_{jt}$  (resp.  $b_{kt}$ ) coefficients.
2. The index sets  $I(\cdot)$  must be mutually exclusive and exhaustive, and hence constitute a partition of the part indices. A unique tool type thus is specified for each part. Tooling is common to multiple parts to the extent that these index sets are not singletons.
3. The fixed cost coefficients  $f_{it}$  are perhaps best interpreted as surrogates for detailed setup costs. The reason is that  $f_{it}$  is incurred when part  $i$  is produced in period  $t$  irrespective of whether this requires a tool changeover (part  $i$ 's tool type may be common to the part run previously), and irrespective of whether more than one machine must simultaneously make part  $i$  in order to achieve the planned production  $X_{it}$ ; to specify setup costs at this higher level of detail would require a major revision of the model that would transport it from the realm of planning to the realm of detailed scheduling. Yet setup costs cannot be ignored entirely because

this tends to cause some of every part to be produced during every period, a situation clearly unacceptable from the production viewpoint. Our solution is to take the  $f_{it}$ 's as empirically weighted average setup costs.

4. Ending inventory level is the only significant terminal condition of the model. A plausible choice is to set  $I_{iT}$  equal to half the desired lot size plus the desired seasonal inventory for part  $i$  at the time in the seasonal cycle corresponding to the end of period  $T$  (based on historical operating experience, insights obtained previously with the help of the model, and managerial judgment).
5. The maximum possible production  $m_{it}$  is the smaller of two limits: the physical limit imposed by full utilization of all available tooling and machines, and the limit on the amount of production that could be absorbed considering the total demand over the planning horizon, specified ending inventory, and current inventory.
6. The profit coefficient  $p_i$  is applied to  $\sum_t X_{it}$  instead of to  $\sum_t S_{it}$ . The rationale for this is that everything made will be sold sooner or later; applying  $p_i$  as indicated avoids the need to value initial inventory  $I_{i0}$  or ending inventory  $I_{iT}$ .
7. The rationale for the policy parameters  $\alpha_{it}$  is that demand levels for different parts may be interdependent: if scarce production resources are allocated only to the most profitable parts, thin product lines and spotty product availability may displease customers and result in lower market share for the profitable items.

Size

For the practical application at hand, problem (1) - (10) has approximately

40,000 continuous variables (I,S,W,X)

12,000 integer variables (Y)

26,000 constraints of type (2),(3),(4),(5),(7).

Problems of this magnitude are generally considered to be far beyond the current state-of-the-art of general mixed integer linear programming.

## 2. SOLUTION BY LAGRANGEAN RELAXATION

Lagrangian relaxation [4] [5] [8] with respect to (3) is an attractive way to generate upper bounds on the optimal value of (1) - (10). The Lagrangian subproblem separates into as many independent simple transportation problems in the  $W$  variables as there are time periods, and as many independent dynamic single-item lot-size problems as there are parts. The original monolith is thereby decomposed into manageable fragments. A good choice for the Lagrangian variables can be obtained efficiently by solving the conventional LP relaxation of (1) - (10), which is equivalent to a single pure network problem. Moreover, Lagrangian relaxation with respect to (3) does not satisfy the Integrality Property defined in [5] and hence is likely to be an improvement over the conventional LP relaxation.

These observations, explained in detail below, are the basis of a solution procedure that has proven to be quite effective.

One can build a branch-and-bound procedure around this Lagrangian relaxation, but it has not proven necessary to do so for the industrial application which stimulated this work. It has been sufficient to generate a feasible solution to (1) - (10) based on the Lagrangian solution. The objective value of this solution has unfailingly been sufficiently close to the upper bound from Lagrangian relaxation that no further refinement has been needed.

A formal description of the solution procedure is now presented.

Step 1 Solve the usual linear programming relaxation of (1) - (10) via the equivalent capacitated network formulation. Denote the associated dual variables for (3) by  $\bar{\lambda}_{jt}$ .

Step 2 Form the Lagrangean relaxation of (1) - (10) with respect to (3) using  $\bar{\lambda}$ . Separate it into independent subproblems in  $W$  for each  $t$  and, for each  $i$ , in the remaining variables.

Solve each of the Lagrangean subproblems by specialized algorithms.

Denote the combined optimal solution to the full Lagrangean relaxation by  $(I^0, S^0, W^0, X^0, Y^0)$  and its optimal value by  $UB$ .

Step 3 Solve (1) - (10) with  $Y$  set "elastically" to  $Y^0$ , that is, (10) is relaxed to  $0 \leq Y_{it} \leq 1$  with  $f_{it}$  set to 0 if  $Y_{it}^0 = 1$  and augmented by a large positive constant if  $Y_{it}^0 = 0$ . Denote the optimal solution to this problem by  $(I', S', W', X', Y')$ . Let  $Y''$  be  $Y'$  with all fractional components rounded up to unity. Form the revised solution  $(I', S', W', X', Y'')$ , and denote its objective value under (1) as  $LB$ . This solution is feasible in (1) - (10) and is within  $UB-LB$  of being optimal. Stop.

Step 1 yields an equivalent capacitated network problem because, when (10) is relaxed to  $0 \leq Y_{it} \leq 1$  for all  $i, t$ , the relation  $Y_{it} = X_{it}/m_{it}$  must hold at optimality for all  $i, t$ . Upon elimination of  $Y$ , (7) becomes redundant, the relaxed version of (10) becomes

$$(10)' \quad 0 \leq X_{it} \leq m_{it}, \text{ all } i, t$$

and (1) can be rewritten as

$$(1)' \quad \underset{I, S, W, X}{\text{MAXIMIZE}} \quad \sum_{i, t} \pi_{it} X_{it} - \sum_{j, k, t} c_{jkt} W_{jkt} - \sum_{i, t=1}^{T-1} h'_{it} I_{it} - H$$

where

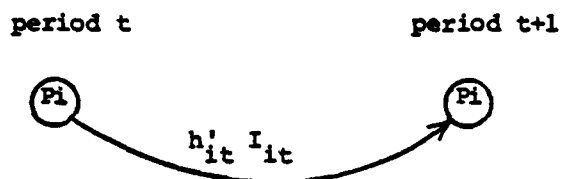
$$(11) \quad \pi_{it} \triangleq p_i - \frac{f_{it}}{m_{it}} \quad \text{for all } i, t$$

$$(12) \quad h'_{it} \triangleq \frac{1}{2} (h_{it} + h_{i, t+1}) \quad \text{for all } i \text{ and } t=1, \dots, T-1$$

$$(13) \quad H \triangleq \frac{1}{2} \sum_i (h_{i1} I_{i0} + h_{iT} I_{iT})$$

It is easy to see that the resulting linear programming problem can be formulated as a minimum cost capacitated network flow problem.

See Figure 1 for an example with 3 parts, 2 tool types, 3 machine groups, 3 time periods,  $I(1) = \{1\}$ ,  $I(2) = \{2,3\}$ ,  $K(1) = \{1,2\}$ , and  $K(2) = \{2,3\}$ . The notational conventions followed in Figure 1 are: the term of (1)' corresponding to each arc is written over the arc (omission means that the unit flow cost is 0), the upper capacity limit of each arc is written under it (omission implies infinite capacity), and the constraint on the net outflow of each node is written under it (omission implies = 0, or strict conservation). The curved arcs between the part nodes are not annotated for lack of room; the typical arc is:



Several variants of the network formulation pictured in Figure 1 are possible.

The Lagrangean relaxation of Step 2 is composed of the following independent subproblems: for each  $t$ ,

$$(R^t) \quad \text{MINIMIZE} \sum_{w \dots t} \sum_{j \in K(j)} (c_{jkt} - \bar{\lambda}_{jt}) w_{jkt}$$

subject to (2), (4), and (8) for fixed  $t$

and, for each  $i$ ,

$$(R_i) \quad \text{MINIMIZE} \sum_{I_{i.}, S_{i.}, X_{i.}, Y_{i.}} \sum_t (\bar{\lambda}_{j(i)t} - p_i) x_{it} + \sum_{t=1}^{T-1} h'_{it} I_{it} + \sum_t f_{it} y_{it}$$

subject to (5), (6), (7), (9), (10) for fixed  $i$



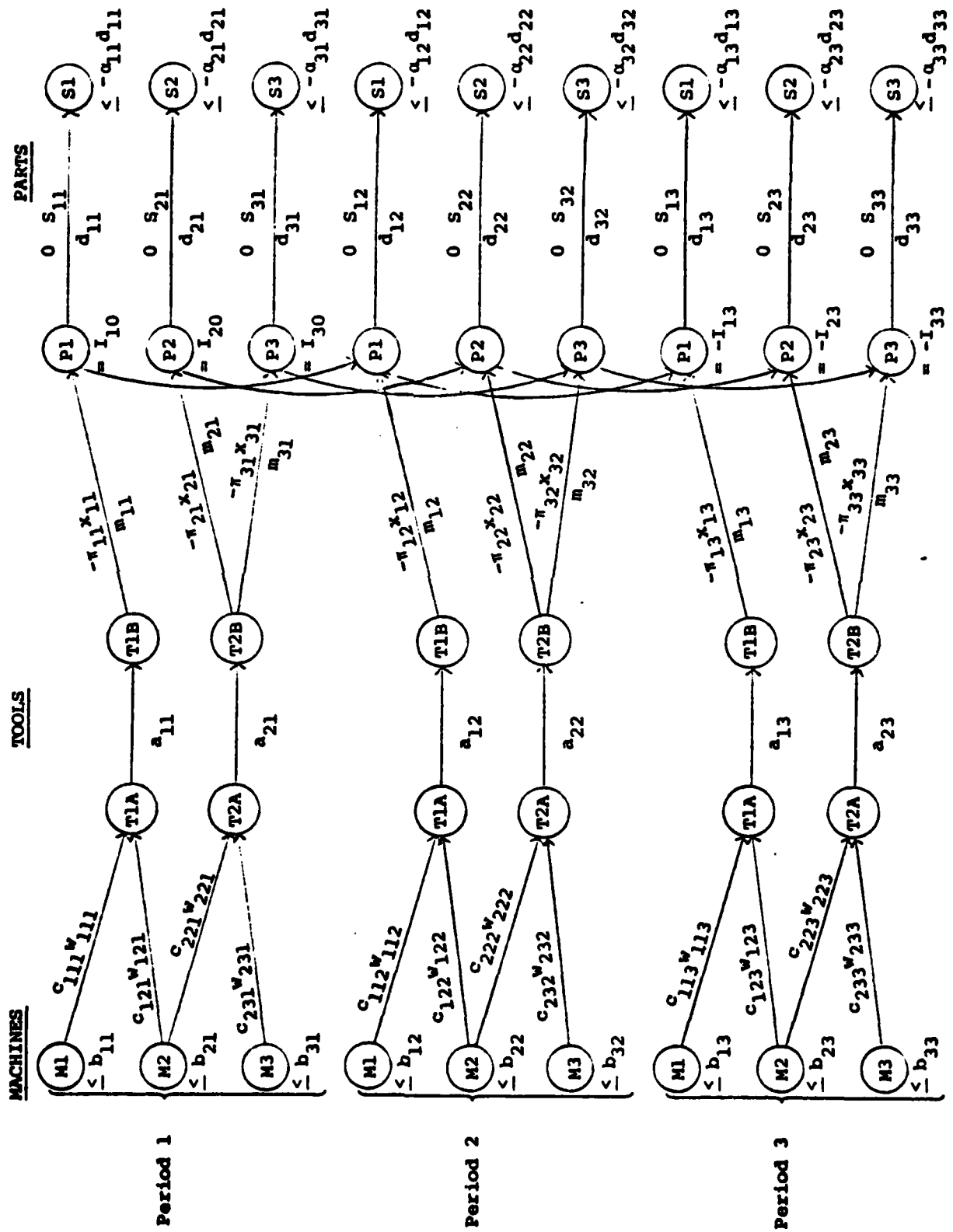


FIGURE 1: SAMPLE EQUIVALENT NETWORK FOR STEP 1

where  $j(i)$  is the index of the tool type required by part  $i$ . Using an obvious notation, the optimal value of the full Lagrangean relaxation of Step 2 is

$$(14) \quad UB \triangleq \sum_t v(R^t) - \sum_i v(R_i) - H.$$

Figures 2 and 3 portray  $(R^t)$  and  $(R_i)$  for the example illustrated in Figure 1. Notational conventions are the same as before except for the arcs leaving node  $Q_i$  in Figure 3: these are dashed to indicate that they are "fixed charge" arcs, with the amount of fixed charge incurred by their use given as the first of the two annotations written over the arcs.

Problem  $(R^t)$  can be converted easily to a simple transportation problem. However, it can be shown using LP duality theory that the W-part of the optimal solution found at Step 1 is necessarily optimal also in these subproblems. No work at all need be performed in connection with these subproblems!

Problem  $(R_i)$  has as many fixed charge arcs as there are time periods. Its special structure invites the development of a specialized solution procedure (e.g., [1] treats a closely related class of dynamic lot-size problems which is a special case of  $(R_i)$ ).

Step 3 yields a problem virtually identical in form to that of Step 1. It can be solved efficiently in the same manner.

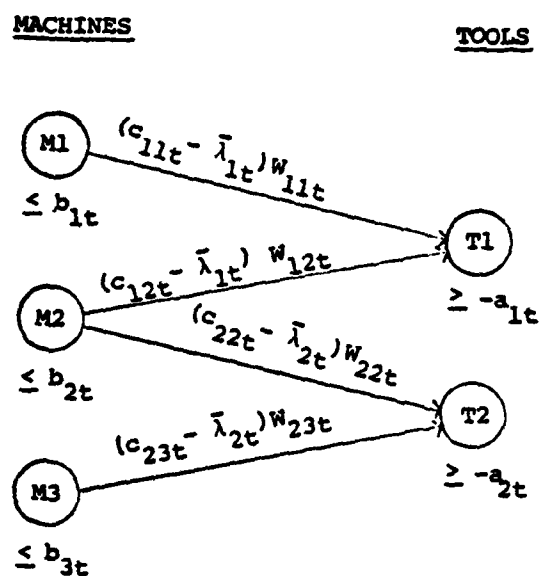
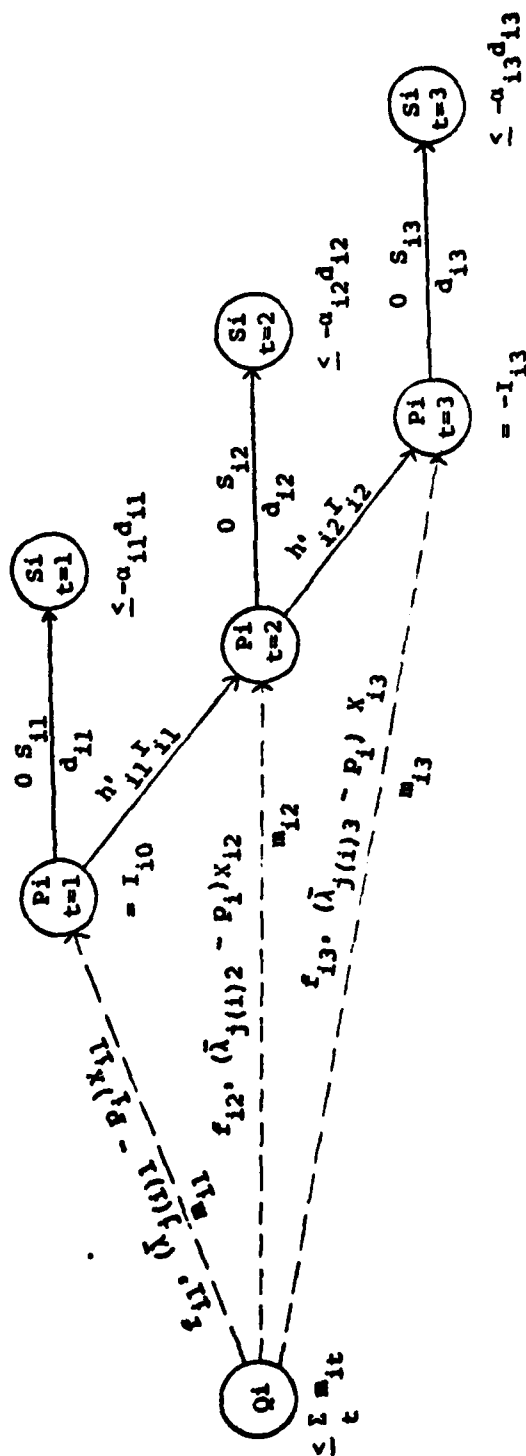


FIGURE 2: SAMPLE EQUIVALENT NETWORK FOR  $(R^t)$

FIGURE 3: SAMPLE EQUIVALENT FIXED CHARGE NETWORK FOR  $(R_1)$

### 3. APPLICATION AND COMPUTATIONAL RESULTS

The model and computational procedure described above have been under development and application for more than two years at both plants of R & G Sloane Manufacturing Company of Sun Valley, California. The following identifications and specializations are appropriate.

| <u>General Model</u>       | <u>Molding Application (main plant)</u>   |
|----------------------------|---|
| parts (i)                  | the top 1000 injection molded fittings (about 92% of all sales volume)                        |
| tool types (j)             | about 90 types of interchangeable mold bases (total mold base census about 130)               |
| machine groups (k)         | about 15 groups of interchangeable injection molding machines (total machine census about 60) |
| tool/machine compatibility | about 480 jk combinations permissible   |
| time periods (t)           | typically the next 12 months  |
| $c_{jkt}, f_{it}, a_{it}$  | taken as independent of t   |

The problem faced by R & G Sloane is a strongly seasonal one; with the bulk of the company's business accounted for by residential plumbing products, demand peaks along with residential construction in the summer months. Since the peak season demand rate exceeds the available capacity of mold bases and machines, constraints (2) and (4) tend to be binding at that time of year (typically, about 20% of the mold base constraints and 80% of the machine constraints are binding in at least three months). Typical relative

magnitudes of the major cost categories associated with an optimal solution are:

|                    |             |
|--------------------|-------------|
| fixed costs        | 14.3        |
| inventory carrying | 16.9        |
| variable operating | <u>68.8</u> |
|                    | 100.0       |

Unfilled demand in most optimal solutions occurs for 2 or 3% of all parts.

#### Computational Implementation

A full scale computational implementation has been carried out for this application. The computer programs are in three modules:

1. data extraction and data base definition
2. problem preprocessing and diagnosis
3. optimization and report writing.

Data extraction primarily involves conversion of current production, marketing, and inventory control operating data to the form required by the model. The data base is organized and generated in sections:

- problem parameters and conditions
- machine group descriptions
- mold bases and their machine compatibility
- part descriptions and demand forecasts.

Preprocessing identifies structural and mathematical inconsistencies in the problem posed, and assists in preliminary diagnosis of critical shortages in equipment availability.

The optimization module solves the capacitated pure networks presented in Steps 1 and 3 with a GNET variant (XNET/Depth) [2]; an advanced starting solution is used which assumes high equipment utilization. The fixed charge problems ( $R_i$ ) are solved in Step 2 with a highly specialized fixed order enumeration algorithm employing GNET/Depth; key elements of this procedure identify and exploit dominant problem features such as mandated maximum production, and permit parametric relaxation of the full enumeration for prohibitively long solution sequences.

The solution reports are presented at several levels of aggregation so as to facilitate managerial interpretation. They display all detailed solution features, estimated opportunity costs for critical mold bases and machines, and an overall analysis of profitability, turnover, and customer service.

#### Computational Results

Approximately 30 runs have been made during the last year. Computational performance has exhibited a high degree of run-to-run stability in terms of the quality of solutions produced and the amount of computer resources expended.

Table 1 summarizes several aspects of performance for a recent typical run of the optimization module. With report writing time included, the total CPU time for this run was 165.5 seconds. The main storage requirement was about one megabyte. Notice that the bound produced by the Lagrangean relaxation is significantly better than the ordinary linear programming relaxation bound. Notice also that the time in Step 2 is smaller than what one might expect; the 12-period fixed charge problems were solved in an average of only .027 seconds each (for comparison, the typical time quoted in [1] for a proper subclass of

|                              | <u>Pivots</u> | <u>IBM 370/168<br/>CPU Seconds</u> | <u>Normalized<br/>Objective<br/>Value</u> |
|------------------------------|---------------|------------------------------------|---|
| <u>Step 1</u>                |               |                                    |   |
| (LP Relaxation)              | 37,933        | 55.0                               | 103.2                                     |
| <u>Step 2</u>                |               |                                    |   |
| (Lagrangean Relaxation)      | 376,241       | 25.7                               | 101.6                                     |
| <u>Step 3</u>                |               |                                    |   |
| (Generate Feasible Solution) | 36,954        | 54.2                               | 100.0                                     |

TABLE 1

Typical Computational Performance  
(953 parts, 92 tool types, 16 machine groups)

of these problems of the same size was 0.25 seconds on an IBM 370/158). For this run, 142 (resp. 10) of the 11,436 binary Y variables changed from value 0 (resp. 1) in Step 2 to value 1 (resp. 0) in Step 3. This shows that the solution to the Lagrangean relaxation of Step 2 required but minor adjustment with respect to the fixed change arcs in order to yield the good feasible solution of Step 3.

The pre-optimization modules required 10 seconds for the run reported in Table 1.

More generally, our experience has been that optimization CPU time for similar sized problems seldom varies more than  $\pm 10\%$ . Computing time is very nearly proportional to the total number of parts. The final optimality tolerance (which was 1.6% in the Table 1 run) tends to become tighter the more tightly capacitated tool and machine availability is; tolerances in



the vicinity of 2/10 of 1% are commonly observed in the most tightly constrained situations. In no case has the tolerance ever exceeded 2%.

#### 4. CONCLUSION

This paper has demonstrated the practical applicability of a procedure based on Lagrangean relaxation to a significant class of integrated production and sales planning models. The particular way in which this procedure is designed thoroughly exploits the recent major advances made for minimum cost network flow problems. Provably good solutions are routinely being obtained in modest computing time to mixed integer linear programs of a size far beyond the capabilities of existing general-purpose mathematical programming systems.

The system is used regularly at R & G Sloane Manufacturing company for production scheduling in the sense that day-to-day scheduling is still performed manually but with the benefit of the system's guidance and predictions of bottlenecks in the future. The integrated nature of the model has made the system valuable as a focal point for coordinating planning activities among the key functional areas of the firm: inventory control, finance, marketing, and production operations. Two specific illustrations are the evaluation of major capital expenditure and interplant equipment transfer opportunities.

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